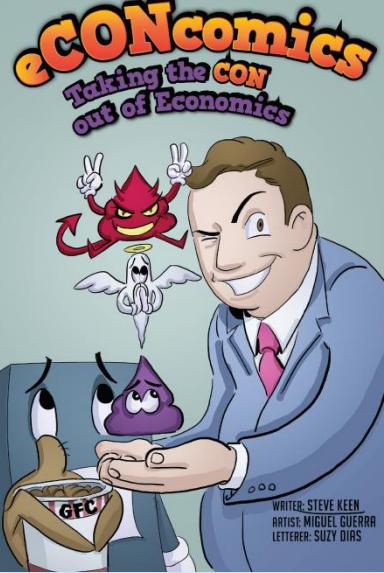
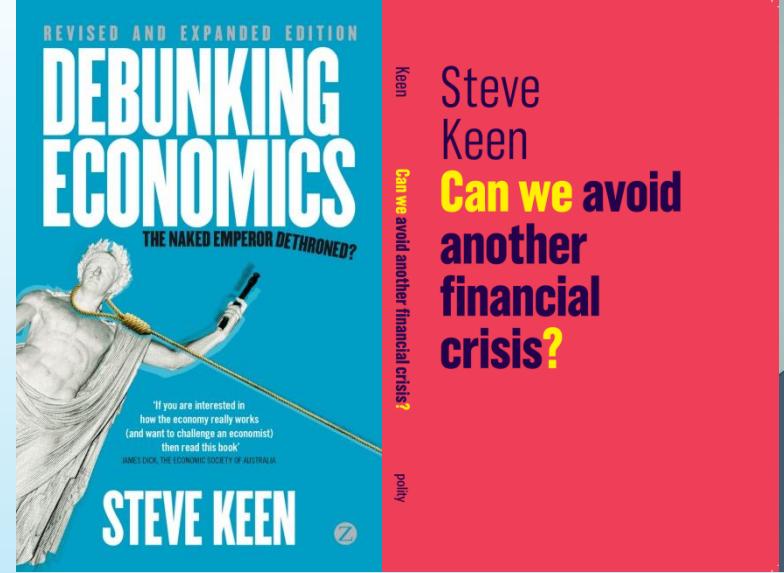




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# The role of energy in production

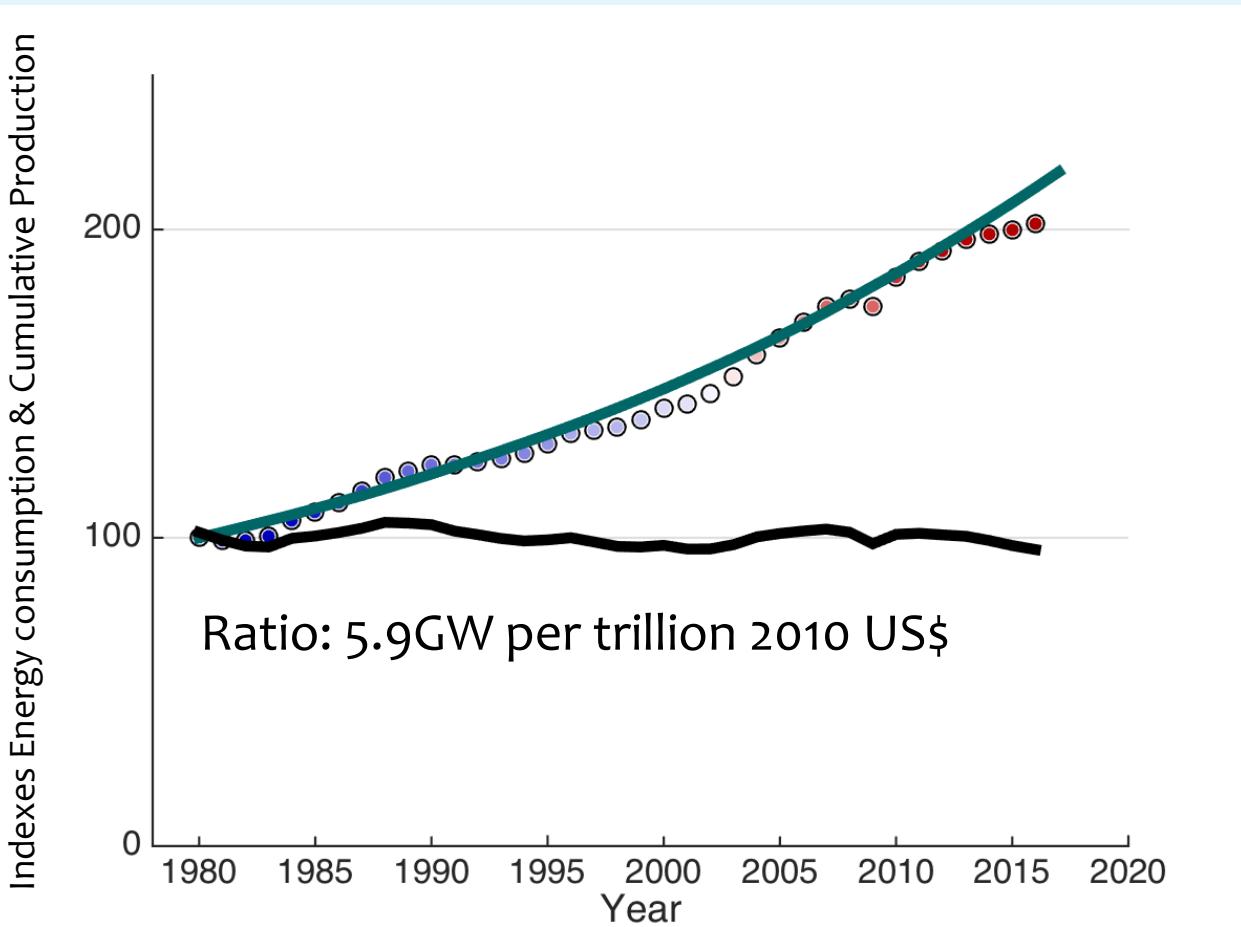
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# The role of energy in production

- Project to develop models of production in which energy plays an essential role
- Two components
  - Revising existing economic models of production (Neoclassical: “Cobb-Douglas”; Post Keynesian: “Leontief”) to include energy in a fundamental way
  - Building on thermodynamics-derive model of economics developed by atmospheric scientist Dr Tim Garret
    - “The Garrett Relation”: thermodynamically derived & empirically verified relationship between energy consumption and wealth → the **integral** of GD



Relation between Cumulative Global Production and Energy consumption: 5.9GW per trillion 2010 US\$. Standard deviation 0.1GW /US\$<sup>12</sup>

# The role of energy in production

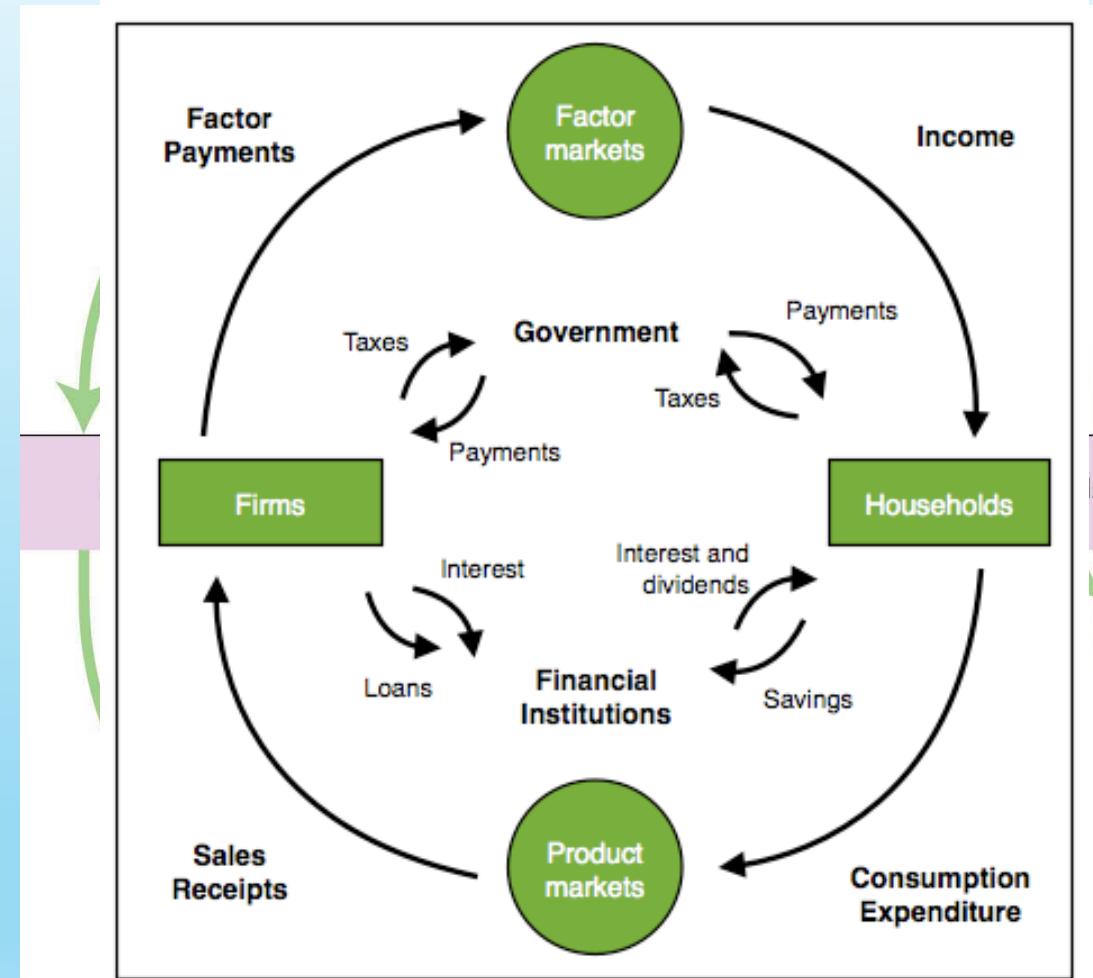
- Research team
  - Tim GarrettA photograph of Tim Garrett, a man with brown hair, wearing a grey and red jacket, standing outdoors with snow-covered mountains in the background.
  - Matheus GrasselliA portrait of Matheus Grasselli, a man with dark hair, wearing a grey blazer over a black shirt, against a plain grey background.
  - Steve KeenA photograph of Steve Keen, an older man with glasses and grey hair, wearing a suit, appearing on a television screen with the text "IT'S THE PRIVATE DEBT, STUPID" and "WE ARE" visible.
- Professor Atmospheric Sciences, Utah
  - Primary research focus the physics of clouds
  - Developed thermodynamic models of civilization growth (Garrett 2011, 2012, 2014 2015)
- Professor of Mathematics, McMaster
  - Primary research focus non-equilibrium monetary macroeconomics
- Honorary Professor, UCL ([ISRS](#))
  - Primary research focus non-equilibrium monetary macroeconomics

# Production, Energy & the Laws of Thermodynamics

- All schools of economic thought model Output as a function of Labour and Capital
  - Neoclassical (Cobb-Douglas):  $Y(t) = A(t) \times L(t)^{1-\alpha} \times K(t)^\alpha$
  - Post Keynesian (Leontief) :  $Y(t) = \min \left( a(t) \times L(t), \frac{K(t)}{v} \right)$
- Energy plays no explicit role in canonical macroeconomic models
- But “production without energy” violates fundamental laws of physics:
  - “The law that entropy always increases, holds, I think, the supreme position among the laws of Nature.
  - If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations — then so much the worse for Maxwell's equations. If it is found to be contradicted by observation — well, these experimentalists do bungle things sometimes.
  - ***But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.***  
*(Eddington, 1928)*

# The environmental blind-spot in economics

- “The Circular Flow Diagram” treats economy in complete isolation from the environment
- There is no “non-economic” input or output
  - All inputs produced by economy itself
  - All output consumed in the economy
- Implies that
  - All inputs can be produced by the economy itself (**violates 1<sup>st</sup> Law of Thermodynamics**)
  - Closed system can produce increasing order over time (**violates 2<sup>nd</sup> Law**)
  - And that production without waste is possible (**violates 2<sup>nd</sup> & 3<sup>rd</sup> Laws**)...

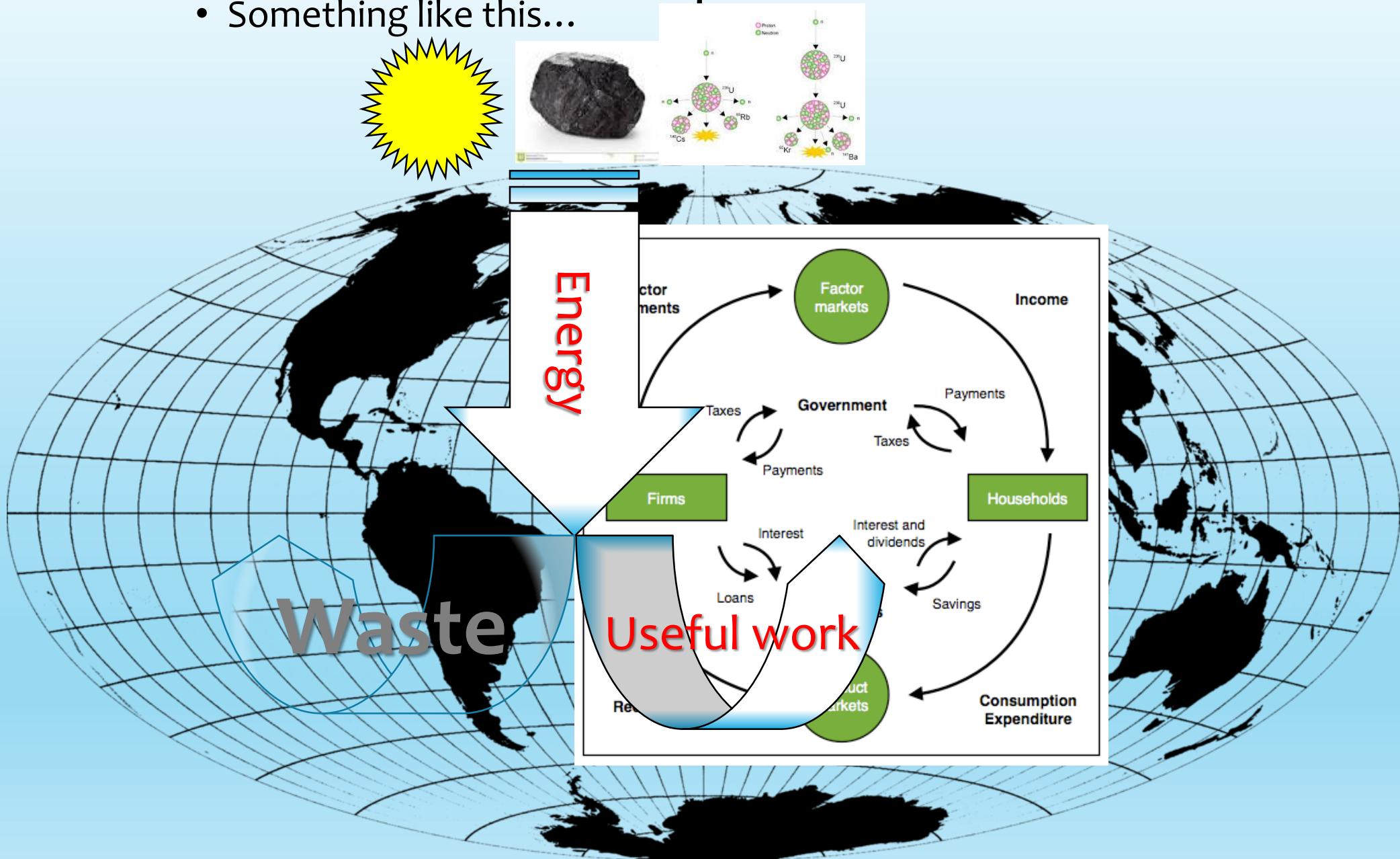


# The environmental blind-spot in economics

- “Circular flow” only possible because we exploit energy already existing in the environment
  - Solar energy; fossil fuels; nuclear energy
    - We cannot “produce” energy
      - This is the “1<sup>st</sup> Law of Thermodynamics”
    - We can only exploit energy that already exists
      - Including converting matter into energy (“nuclear power”)
- Exploiting that energy **necessarily** generates waste
  - Waste can be minimised but not eliminated
    - These are the 2<sup>nd</sup> and 3<sup>rd</sup> Laws of Thermodynamics
- Minimum modification needed to make “circular flow” diagram realistic is to add
  - Energy inputs from the (not man-made) environment
  - Waste injected back into the environment...

# The environmental blind-spot in economics

- Something like this...



# The environmental blind-spot in economics

- Why can't we
  - Produce without using already existing energy?
  - Produce without creating waste?
- The “Laws of Thermodynamics”
  - Empirically absolutely true rules about energy and work
  - Discovered in late 19<sup>th</sup> century
    - After Marx had developed the “Labour theory of value”
    - As Neoclassical economics started to develop
      - Neoclassical economics influenced by physics from 20 years earlier, but not by thermodynamics
- Complicated rules require advanced maths to really understand
- Basics are best conveyed by a mathematics joke...

# The environmental blind-spot in economics

- The “joke” version of the Laws of Thermodynamics (by Alan Ginsberg)
  1. You can't win
  2. You can't break even
  3. You can't leave the game...
  - Slightly more informative version:
    1. You can't win, you can only break even.
    2. You can only break even at absolute zero.
    3. You can never reach absolute zero.
- Translation:
  1. Energy can neither be created nor destroyed
    - a. Only its form can be changed
  2. The maximum amount of energy you can use to perform work is limited by the background temperature in which the work is done
    - a. Only if the background temperature is Absolute Zero can you turn all available energy into work
  3. There is no place in the Universe that is at Absolute Zero

# The environmental blind-spot in economics

- It gets worse... The 2<sup>nd</sup> Law of Thermodynamics:
  - Heat must be exchanged with an “external system”
    - Otherwise no energy can be extracted, because...
    - **A closed system degrades to uniform temperature over time**
- Power generation works because waste heat dumped in environment:
  - Waste is inevitable
  - Without waste there is no useful work

If this bit were completely isolated from the environment...

Because this dumps waste heat into the environment

# Production with Energy

- At a time when energy/ecological concerns are paramount, existing economic theories (Post Keynesian as well as Neoclassical) have nothing **fundamental** to say about either
- Can we make economic models of production consistent with thermodynamics?
- A simple insight: “**Labour without energy is a corpse; Capital without energy is a sculpture**” (Keen, Ayres & Standish 2019, p. 41)
  - Labour & Capital as means to convert energy into useful work:
    - Use  $E_L$  &  $E_K$  for the energy inputs to workers & machines
    - Replace  $L(t)$  and  $K(t)$  with  $L(E_L(t))$  &  $K(E_K(t))$
- Acknowledges essential role of energy in production
  - No energy in, no output out
- Acknowledges unavoidability of waste
  - Conversion of energy to useful work must be less than 100% efficient (2<sup>nd</sup> Law)

# Production with Energy

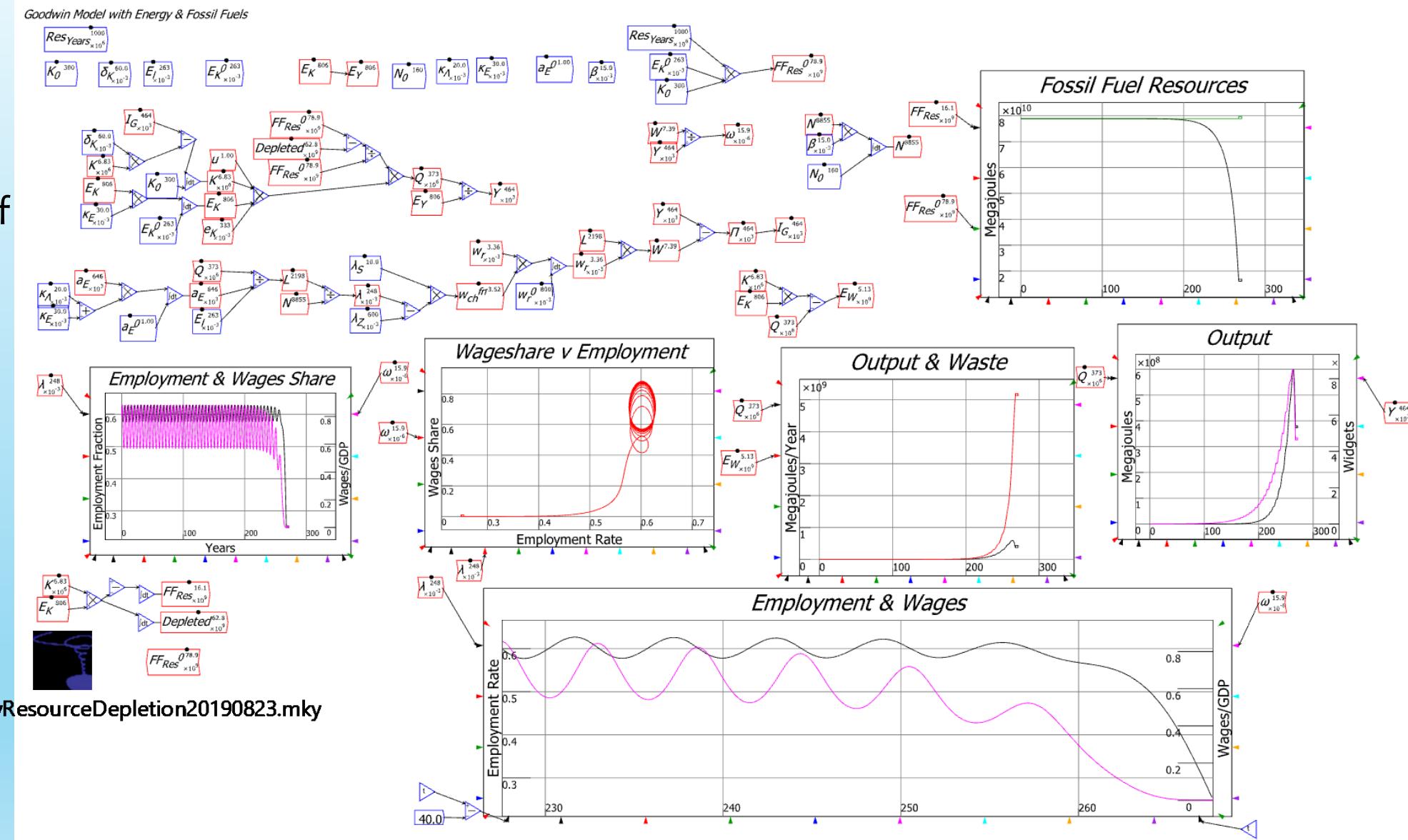
- Introduce  $Q$  as output in terms of energy:
  - Cobb-Douglas (CDPF):  $Q = L(E_\Lambda)^{1-\alpha} \times K(E_K)^\alpha$
- Simplest way to define  $E_\Lambda$  and  $E_K$ 
  - $L(E_\Lambda(t)) = L \times E_L \times e_L$
  - $K(E_K(t)) = K \times E_K \times e_K$
- Results when fed into Cobb-Douglas & Leontief functions:
  - (1) “Solow Residual”/”Total Factor Productivity” is actually the useful work output of the representative machine
  - (2) Exponent for exergy (useful work) of machine is the same as for capital
  - (3) A much larger exponent is justified for capital & machine exergy
  - (4) “Capital-Output ratio” is actually efficiency of conversion of machine energy input into useful work
  - (5) “Labor productivity” is the ratio of machine to human useful work

# Production with Energy: Goodwin model

- Introducing energy into production models solves conundrums in existing theory
  - Puzzling “Solow Residual” is the energy consumption of machinery
    - Rise in this over time sensibly explains most of growth in GDP
  - “Total Factor Productivity” is the same thing: energy consumption of machinery
- Enables economic models to be fundamentally linked to ecology
  - Energy availability/cost necessarily affect economic performance
  - Rising GDP necessarily causes biosphere pollution ( $\text{CO}_2$  simply major form)
  - Declining EROEI (“Energy return on energy invested”) will reduce efficiency of conversion of energy into useful work  $e_K$
- An illustration: Goodwin growth cycle model with energy
  - Illustration of potential feedbacks:
    - **Hypothetical** effect of exhaustion of fossil fuel reserves on GDP
    - Accumulation of waste as necessary outcome of production

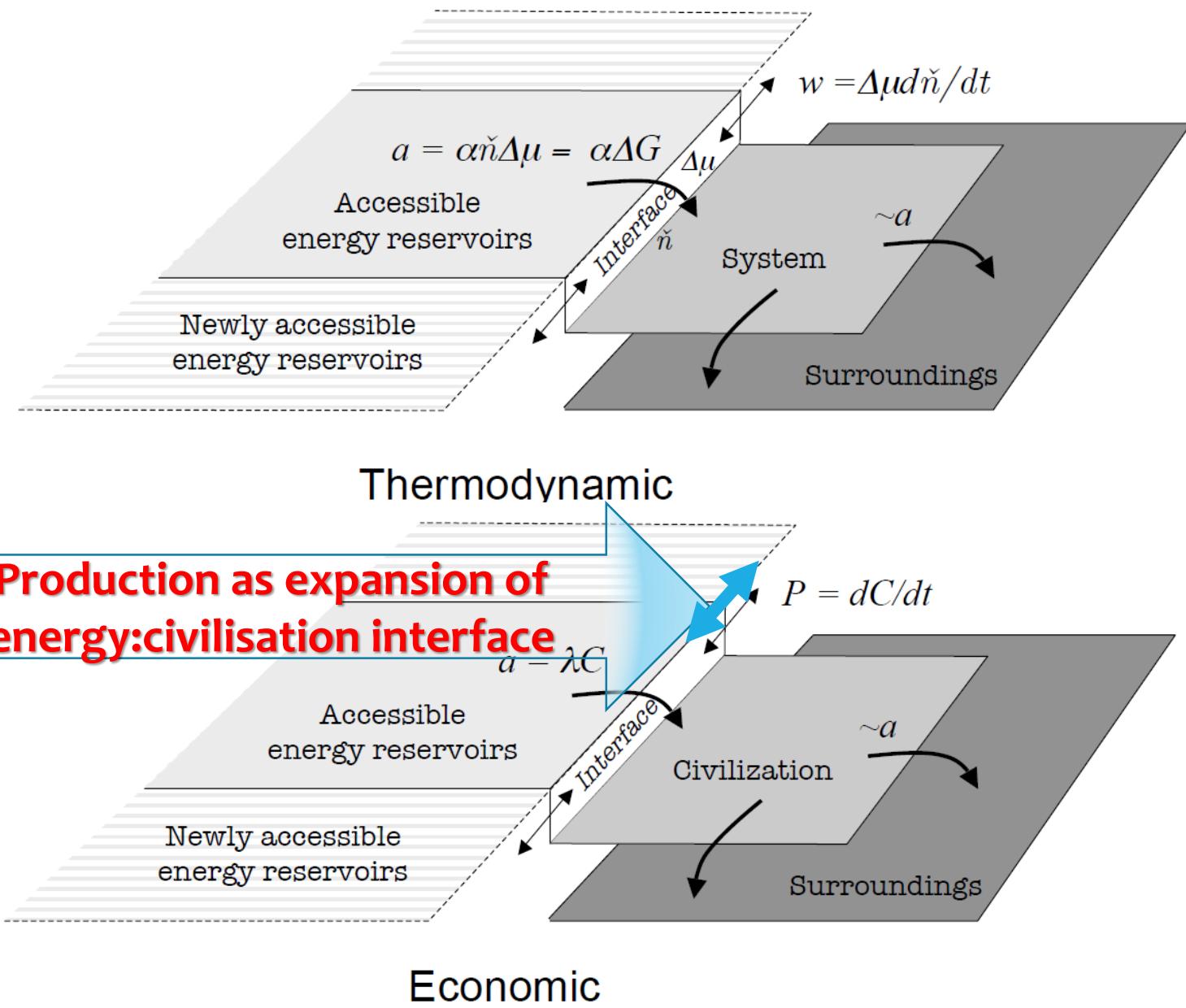
# Production with Energy: Goodwin model

- Illustrative model only: production with exhaustion of fossil fuel energy reserves



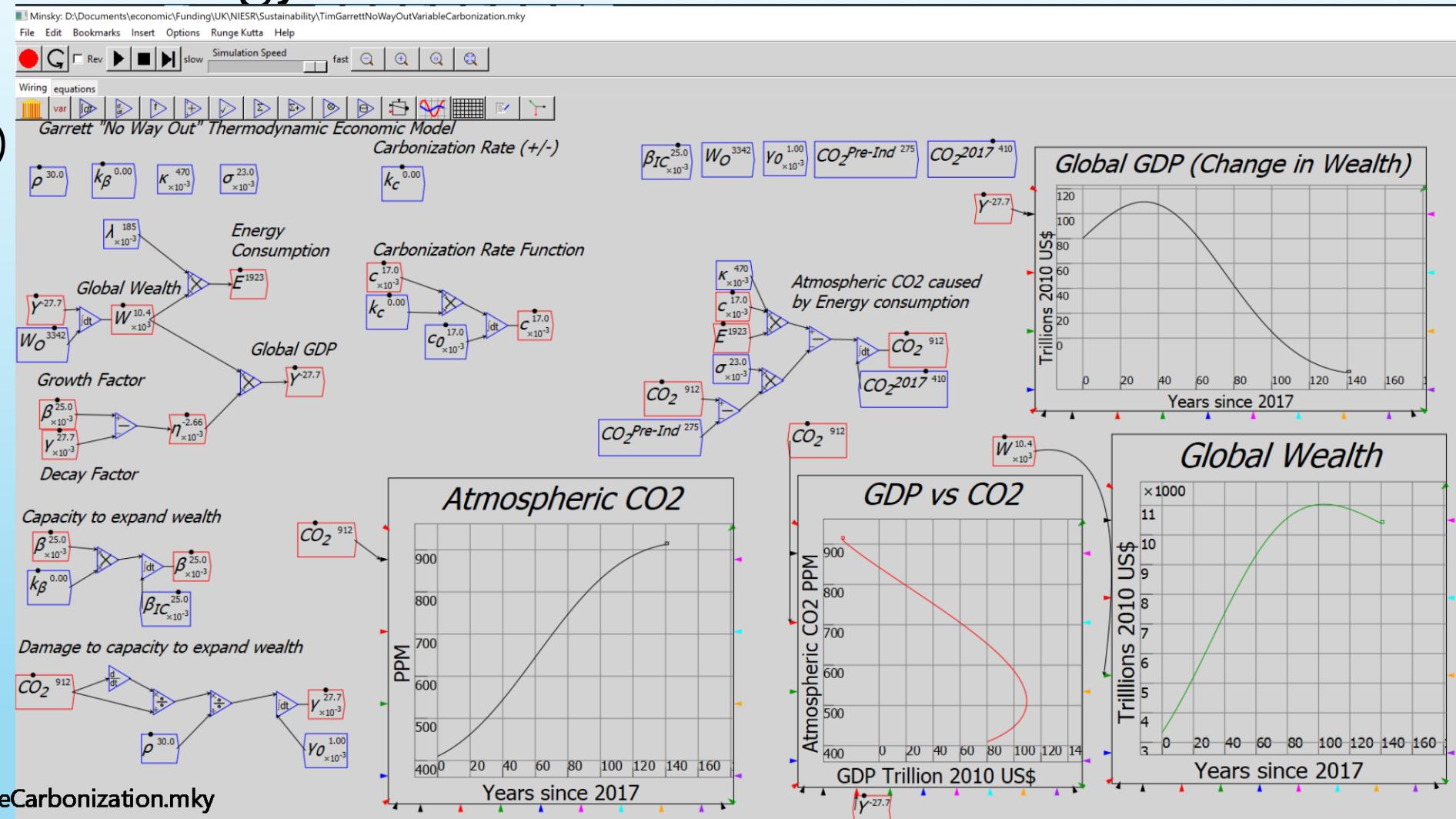
# Production with Energy: The Garrett Relation

- Derived by analogy to formation of natural energy dissipating structures (raindrops, snowflakes, cyclone) in response to an energy gradient, dissipating the energy into the surrounding environment
- Human society similarly exploits an energy gradient (potential energy of fossil fuels) into a structure (civilisation), dissipating the energy into the environment



# Production with Energy: The Garrett Relation

- Derives a link between wealth (integral of GDP) and energy
- Therefore between GDP (change in wealth) and change in energy



TimGarrettNoWayOutVariableCarbonization.mky

- Confronting hypothetical: without significant de-carbonisation, future change in wealth will be negative

# Future work

- ***Within existing £60K budget***
  - Expand model to include matter as well as energy inputs
    - Energy primarily used to transform matter from useless to useful forms
    - Waste matter more important than waste energy per se
      - Without waste matter, waste energy would radiate into space
      - With waste matter,  $CO_2 \uparrow$ , other wastes, degrade productive capacity
  - Reconcile Keen Energy → GDP relation with Garrett Energy → Wealth relation
  - (Possibly): Multi-sectoral models of energy inputs (Fossil vs Renewable/Non-Fossil) and uses (multiple commodities/multiple forms of waste)
- ***Beyond this project***
  - Develop non-equilibrium thermodynamically valid integrated models of the economy with finance and ecology, & feedbacks between the systems (unlike existing Ramsey-based IAMs)

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# Cobb-Douglas's apparent fit to income distribution data

- Appeal of CDPF has been apparent confirmation of marginal product distribution
  - “aggregate production functions apparently work nevertheless and do so in a way which is *prima facie* not easy to explain...
  - In its simplest form, this puzzle is set by a remark which Solow once made to me that, had Douglas found labor's share to be 25 per cent and capital's 75 per cent instead of the other way around, we would not now be discussing aggregate production functions.
  - If the fact that estimated aggregate production functions explain wages fairly well is a statistical artifact, then it is certainly not an obvious one.” (Fisher 1971)
- True: it does take thought to uncover the statistical artifact:
  - Shaikh, A. (1974). “Laws of Production and Laws of Algebra: The Humbug Production Function.” *Review of Economics and Statistics* 56 (1): 115-120.
    - See also McCombie (2000) & Felipe and Adams (2005)...

# Cobb-Douglas's apparent fit to income distribution data

- Define  $Y = \text{Wages} + \text{Profits}$ , Assume uniform wage  $w$  and rate of return on capital  $r$ 
  - $Y = w \cdot L + r \cdot K$
- Differentiate with respect to time, divide by  $Y$ , multiply terms by  $\frac{L}{L}, \frac{w}{w}, \frac{K}{K}$  or  $\frac{r}{r}$ 
  - $\frac{1}{Y} \cdot \frac{dY}{dt} = \frac{1}{Y} \cdot \left( \frac{L}{L} \cdot w \cdot \frac{d}{dt} L + \frac{w}{w} \cdot L \cdot \frac{d}{dt} w + \frac{K}{K} \cdot r \cdot \frac{d}{dt} K + \frac{r}{r} \cdot K \cdot \frac{d}{dt} r \right)$
- Rearrange into income shares & rates of change:
  - $\frac{1}{Y} \cdot \frac{dY}{dt} = \left( \frac{w \cdot L}{Y} \cdot \left( \frac{1}{L} \cdot \frac{d}{dt} L \right) + \frac{w \cdot L}{Y} \cdot \left( \frac{1}{w} \cdot \frac{d}{dt} w \right) + \frac{r \cdot K}{Y} \cdot \left( \frac{1}{K} \cdot \frac{d}{dt} K \right) + \frac{r \cdot K}{Y} \cdot \left( \frac{1}{r} \cdot \frac{d}{dt} r \right) \right)$
- Substitute  $\frac{r \cdot K}{Y} = \alpha$ ,  $\frac{w \cdot L}{Y} = (1 - \alpha)$ , assume (relatively) constant, restate using logs:
  - $\frac{d}{dt} \ln(Y) = \left( (1 - \alpha) \cdot \frac{d}{dt} \ln(L) + (1 - \alpha) \cdot \frac{d}{dt} \ln(w) + \alpha \cdot \frac{d}{dt} \ln(K) + \alpha \cdot \frac{d}{dt} \ln(r) \right)$
- Integrate:  $\ln(Y) = ((1 - \alpha) \cdot \ln(L) + (1 - \alpha) \cdot \ln(w) + \alpha \cdot \ln(K) + \alpha \cdot \ln(r))$
- Take exponentials:  $Y = w^{1-\alpha} \cdot r^\alpha \cdot L^{1-\alpha} \cdot K^\alpha$ 
  - Voila the “Cobb-Douglas Production Function” (with  $A = w^{1-\alpha} \cdot r^\alpha$ )

# The environmental blind-spot in economics

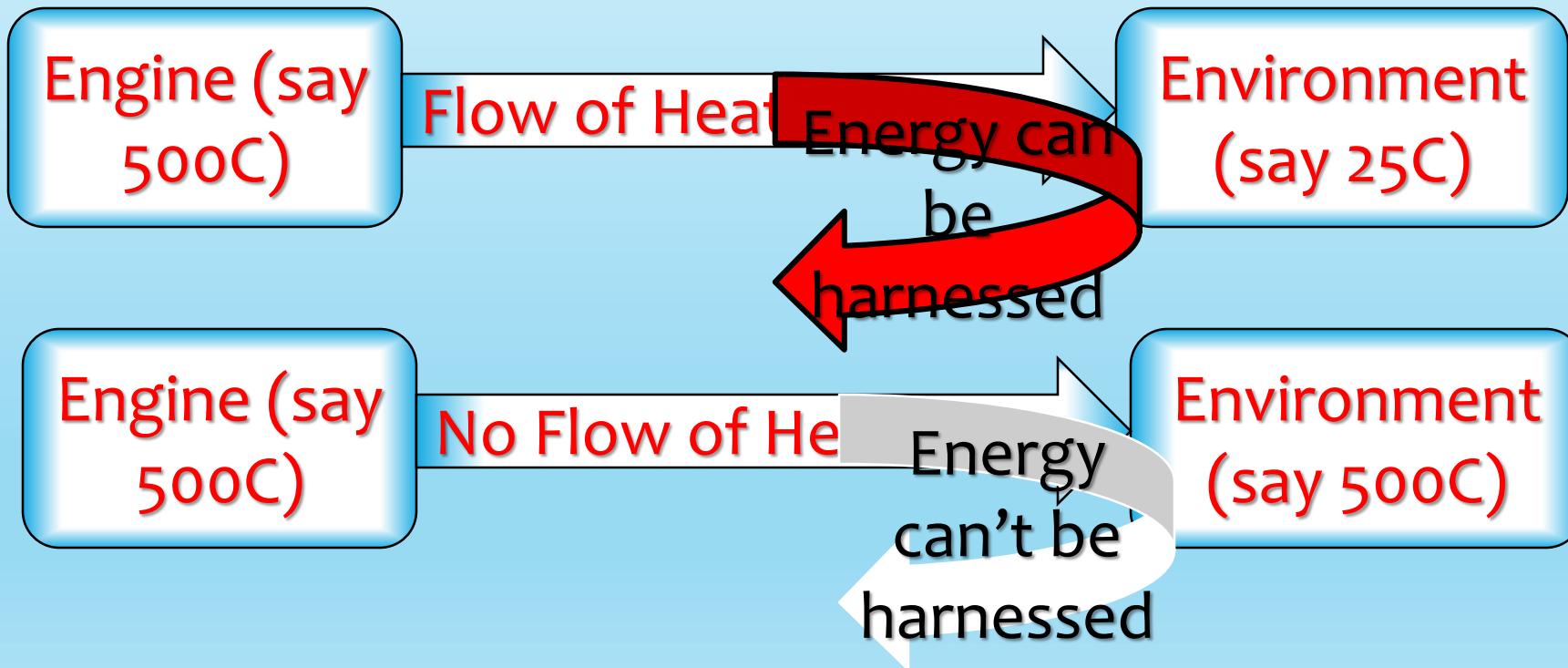
- Why wasted energy is inevitable...
- Using energy requires turning potential energy into work
  - E.g., using a waterfall to turn a mill...



- You can't generate power unless there is a gap between where the water starts and where it ends
- Same thing for energy in general: maximum you can extract depends on gap between energy of source and where it is used...

# The environmental blind-spot in economics

- Waterfall can be source of power if height > 0
- Same idea for energy in general
  - Work can be done if energy of source greater than surroundings
  - Basic idea is a “heat pump”
    - Work can be done if heat of “source” (Heat of engine)
    - Greater than heat of “sink” (Heat of surrounding air)



# Slides removed for length

- Characterizing the components
  - $L$  &  $K$  same as existing production functions
  - $E_L$  (energy consumed by an unskilled worker) has risen exponentially over time
  - But useful work capacity of unskilled labour is unchanged
    - Define  $E_l = E_L \times e_L = \text{constant}$  energy output of unskilled labour
  - $E_K$  has risen exponentially over time.  $E_K = E_{K0} \times e^{\kappa_E \times t}$
  - $e_K$  bounded  $0 < e_K \ll 1$

# Production with Energy: Cobb-Douglas

- Start with
  - $Q = L(E_L)^{1-\alpha} \times K(E_K)^\alpha$
- Substitute  $E_L = E_l$ ,  $E_K = K \times E_K \times e_K$ 
  - $Q = (L \times E_l)^{1-\alpha} \times (K \times E_K \times e_K)^\alpha$
- Gather terms
  - $Q(t) = E_l^{1-\alpha}$  
  - Original CDPF
- Explanation for “Solow Residual”
  - A in modern CDPF is not “total factor productivity”, but “energy output of the representative machine”
- Including energy raises the issue of the value of exponent  $\alpha$ 
  - $Q = (L \times E_l)^{1-\alpha} \times (K \times E_K \times e_K)^\alpha$
  - Cobb-Douglas set  $\alpha = 0.25$
  - Most models today use  $\alpha = 0.3$
  - Why should the much lower energy input of labour have a much higher exponent?

# Production with Energy: Cobb-Douglas

- Mankiw's cross-country data logic for  $\alpha \geq 0.67$  (Mankiw 1995)
- "An increase in the capital share,  $\alpha$ , from one-third to two-thirds raises  $\alpha/(1 - \alpha)$  in the first equation from one-half to two. Income now moves proportionately twice as much as the rate of saving, rather than half as much... the model can now explain variation in income of the magnitude observed..."
- the model now predicts the rate of convergence estimated in the empirical literature...
- predicted return differential falls from a multiple of 100 [ $10^2$ ] to a multiple of 3.16 [ $10^{\frac{1}{2}}$ ]...
- each of the three problems with the neoclassical model of growth would disappear if the capital share were much higher than is conventionally understood. As a theoretical matter, this resolution is attractive for its parsimony: it solves three serious problems by changing the value of one parameter." (pp. 289-92)
  - But how to justify it?...

# Production with Energy: Cobb-Douglas

- Mankiw's rationalizations for  $\alpha \geq 0.67$ :
  - "One way to raise the capital share above one-third is to argue that there are positive externalities to capital..."
  - A second argument for a larger capital share posits that capital is a much broader concept than is suggested by the national income accounts... *human capital* includes both schooling and on-the-job training." (pp. 292-294)
- Our first proposal for a higher  $\alpha$ :
  - Energy output of machinery has the same exponent as K
    - $$Q = C \times (E_K \times e_K)^\alpha \times L^{1-\alpha} \times K^\alpha$$
  - Higher value for  $\alpha$  recognises the essential role of energy in production

# Production with Energy: Leontief

- Same exercise with Leontief Production Function (with capacity utilization  $u$ )

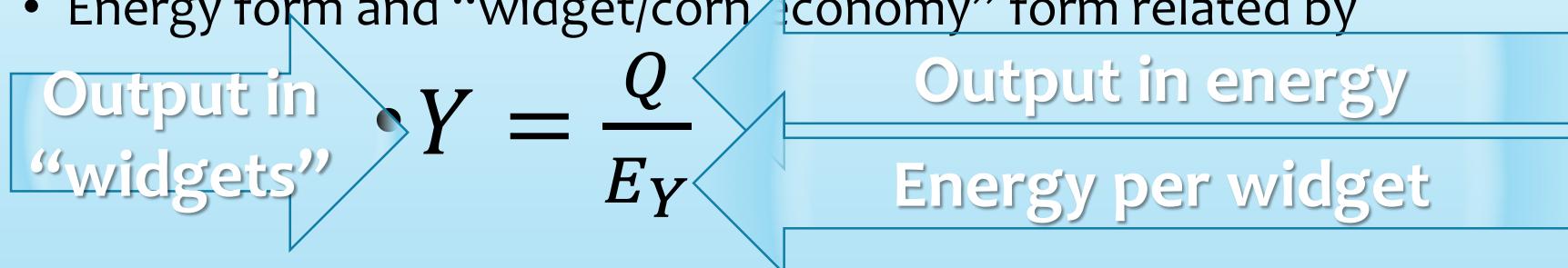
- $Y = u \frac{K}{\nu} = a \times L$

- Introduce energy form  $Q(t) = u \times K(E_K) = u \times K \times E_K \times e_K$

- Energy form and “widget/corn economy” form related by

$$Y = \frac{Q}{E_Y}$$

Output in “widgets”      Output in energy  
Energy per widget



- Cancelling common terms leads to an expression in  $\nu, e_K, E_K$  and  $E_Y$

$$\bullet \frac{1}{\nu} \frac{\text{Energy scalar}}{\text{Scalar Energy}}$$

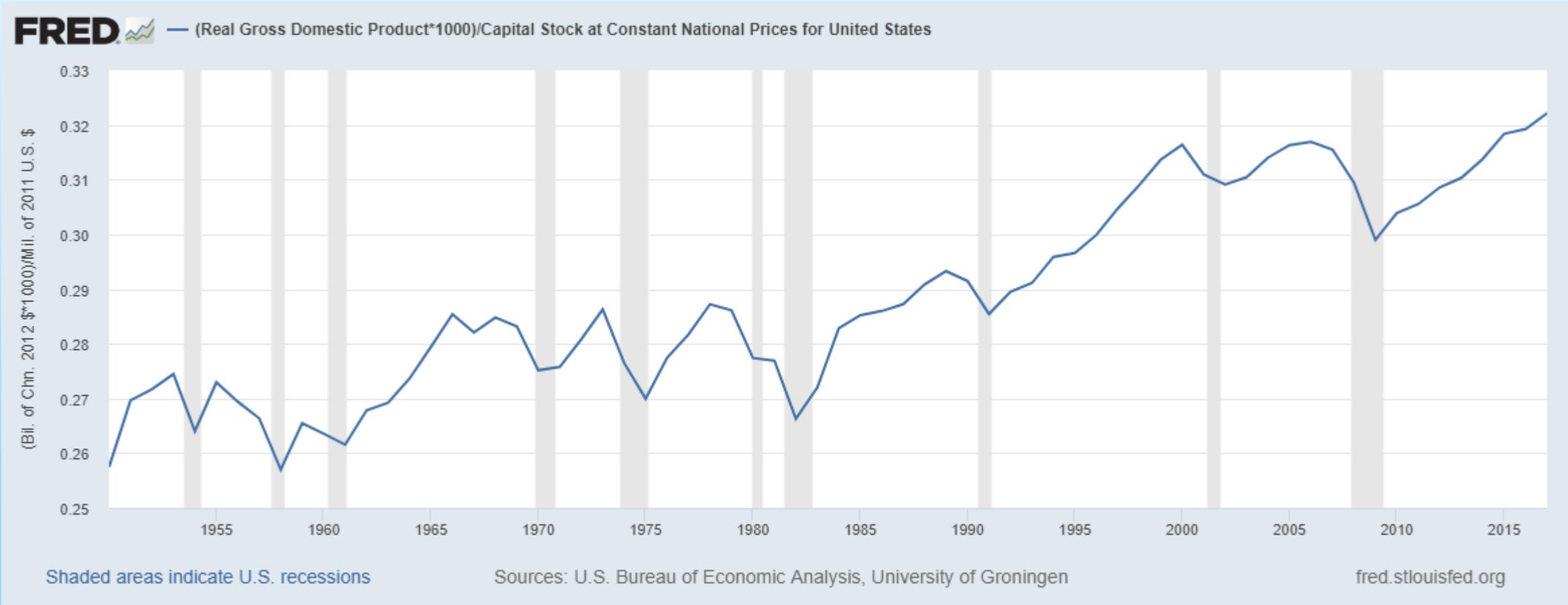
- Equate terms with the same dimensions

- “Energy per widget” term  $E_Y$  equals energy consumption of machinery  $E_K$

- “Capital:output ratio”  $\nu$  equals energy to useful work efficiency of machinery  $e_K$

# Production with Energy: Cobb-Douglas & Leontief

- “Capital output ratio”  $\nu$  is actually the inverse of efficiency with which machines turn energy into useful work



- Declining  $\nu$  reflects **rising** efficiency of manufacturing conversion of energy into useful work